NORDISKT SYMPOSIUM I KOMBINATORIK II
Hässelby Slott, 2 - 4 maj 1984

Program

2 maj

	Lunch	12 - 13	
		13.30 - 15	Astola/Helleseth/Kløve
	Kaffe	15 - 15.30	
		15.30 - 17	Rødseth/Tietäväinen/Tverberg
	Middag	17.30 - 19	

3 maj

	Frukost	8 - 8.30	
		9 - 10	Schrijver
	Kaffe	10 - 10.30	
		10.30 - 11.40	Eklund/Björner
	Lunch	12 - 13	
		13.30 - 15	Nielsen/~~Lindström~~ Frank/Wasén
	Kaffe	15 - 15.30	
		15.30 - 17	Thomassen/Brøndsted/Vestergaard
	Middag	17.30 - 19	

4 maj

	Frukost	8 - 8.30	
		9 - 10 (8.30 - 9)	Lindström
			Aspvall/Proskurowski
	Kaffe	10 - 10.30	
		10.30 - 11.40	Andersen/Häggkvist
	Lunch	12 - 13	
		13.30 - 15 (14.30)	Brouwer/Lovász
	Kaffe	15 - 15.30	
		~~15.30 - 17~~	~~Frank/Tverberg~~

Föredrag vid Nordiskt Symposium i Kombinatorik 2 - 4 maj 1984

Andersen:	Komplettering af partielle symmetriske latinska kvadrater og kantfarvede fuldstaendige grafer
Aspvall:	Matroids, linear inequalities, and flow-with-gain networks
Astola:	On the existence of certain perfect AN-codes
Brøndsted:	Oriented graphs associated with convex polytopes
Björner:	Lexicographically shellable partially ordered sets
Brouwer:	Subgraphs of graphs of Coxeter or Lie type
Eklund:	A comparison of lattice-theoretic approaches to fuzzy topology
Frank:	Entropies of random graphs
Helleseth:	On the covering radius of cyclic linear codes and arithmetic codes
Häggkvist:	Graph decomposition
Kløve:	Generalization of the Korzik bound
Lindström:	On harmonic conjugates in algebraic matroids
Lovász:	Some relaxations of vertex packings in graphs
Nielsen:	Determination of transfer functions for a class of electrical networks by a graph theoretic algorithm
Proskurowski:	Characterization and recognition of partial k-trees
Rødseth:	On sums of sets of residue classes
Schrijver:	On polyhedral combinatorics
Thomassen:	Directed graphs of large girth
Tietäväinen:	Character sums and block codes
Tverberg:	On Schmerl's effective version of Brooks' theorem
Wasén:	Tolerance relations imposed on set algebras
Vestergaard:	Cycles in graphs

NORDISKT SYMPOSIUM I KOMBINATORIK

Hässelby Slott, 2 - 4 maj 1984

S. Alm, Stockholm

L.D. Andersen, Aarhus

S. Arnborg, Stockholm

B. Aspvall, Bergen

J. Astola, Lappeenranta

J. Backelin, Stockholm

A. Björner, Stockholm

A. Brouwer m fam., Amsterdam

A. Brøndsted, København

A. Chetwynd,

P. Eklund, Åbo

O. Frank, Lund

T. Helleseth, Oslo

T. Hellgren, Stockholm

R. Häggkvist, Stockholm

T. Kløve, Bergen

R. Langborger, Stockholm

B. Lindström, Stockholm

L. Lovász, Budapest

F. Nielsen, København

A. Proskurowski, Stockholm

Ø. Rødseth m fru, Stavanger

A. Schrijver, Tilburg

C. Thomassen, København

A. Tietäväinen, Turku

H. Tverberg, Bergen

P. Vaderlind, Stockholm

R. Wasén, Linköping

P.D. Vestergaard, Aalborg

COMPLETING PARTIAL SYMMETRIC LATIN SQUARES
AND PARTIALLY EDGE-COLOURED COMPLETE GRAPHS

Lars Døvling Andersen

I shall discuss a theorem which is analogous to the Evans conjecture for latin squares (now a theorem). The new result is about symmetric latin squares. It states that if at most $n-1$ cells are symmetrically filled in a partial latin square of side n, and if the diagonal is admissible, then it can be completed to a symmetric latin square of side n. I shall also characterize those situations with n and $n+1$ cells filled where completion is not possible.

From these results it is possible to deduce other about completion of partial edge-colourings of complete graphs K_n with approximately $\frac{1}{2}n$ edges coloured.

Keywords: graph reductions, confluent reductions, k-trees

CHARACTERIZATION AND RECOGNITION OF PARTIAL k-TREES †

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ABSTRACT

Our interest in the class of k-trees and their partial subgraphs is motivated by some practical questions about reliability of communication networks in the presence of constrained line- and site-failures, and about complexity of queries in a data base system: We have found a set of confluent reductions on graphs such that any graph can be reduced to the empty graph if and only if it is a partial 3-tree. This set of reductions yields a polynomial time algorithm for deciding if a given graph has a decomposition into 4-vertex graphs no two of which share more than 3 vertices (and finding such a decomposition when it exists). This generalizes the previously known recognition algorithm for partial 2-trees of Wald and Colbourn's.

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Matroids, linear inequalities, and flow-with-gains networks

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Abstract

Linear inequality systems with at most two variables per inequality arise in different areas, for example, flow-with-gains networks and mechanical verification systems. The computational complexity of such inequality systems have been studied by several researchers during the past seven years. We examine some of the ideas used and try to shed light on why the approaches have led to efficient algorithms.

We start by presenting a version of Farkas´ Lemma using matriod terminology. We then study the implication for inequality systems with at most two variables per inequality. Bidirected graphs and voltage-graphic matroids are examples of familiar structures we encounter. Finally, we show how the efficient algorithms employ binary search techniques to check whether a feasible solution exists.

ON THE EXISTENCE OF CERTAIN PERFECT AN-CODES

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Abstract

Denote by Z_M the ring of integers $0,1,\ldots,M-1 \pmod{M}$ and by $|a|$ the least nonnegative residue of a (mod M). The modified binary form of an integer N is the expression

$$(1) \qquad N = a_n 2^n + \ldots + a_i 2^i + \ldots + a_1 2 + a_0 ,$$

where $n \in Z$, $n \geq 0$ and $a_i \in \{-1,0,1\}$ $(0 = 1,\ldots,n)$. The form (1) is minimal if the number of nonzero coefficients is minimal. The arithmetic weight W(N) of an integer N is the number of nonzero coefficients in a minimal form (1) of N. The modular wieght $W_M(N)$ of an integer $N \in Z_M$ is $\min\{W(N),W(M-N)\}$ (there is also an other definition [2]). The modular distance $D_M(N_1,N_2)$ between N_1 and N_2 is $W_M(|N_1-N_2|)$. Recently Ernwall [1] has completely characterized the moduli M for which D_M is a metric.
Let M = AB and M such that D_M is a metric. An AN-code in the ring Z_M is

$$C = \{AN | N \in Z,\ 0 \leq N < B\} .$$

If every integer $0,1,\ldots,M-1$ has modular distance 0 or 1 to exactly one codeword $\in C$ then C is called perfect. It is well known that there are perfect arithmetic codes when $M = 2^n \pm 1$. We shall investigate the existence of such codes in the other cases in which D_M is a metric.

[1] Ernvall, S., When does the modular distance induce a metric in the binary case. IEEE Trans IT 28 No 4, July 1982.

[2] van Lint, J.H., Introduction to Coding Theory. Springer-Verlag, New York. Heidelberg, Berlin, 1982.

ANDERS BJÖRNER

Lexicographically shellable partially ordered sets

Abstract: The lecture will introduce and exemplify the notion of lexicographically shellable posets and describe some of its properties.

Oriented graphs associated with convex polytopes

Arne Brøndsted

Let P be a simple polytope in R^d. "Almost" every $w \in R^d$ induces a natural orientation of the edges of P, namely in the direction of w. (One must require that w is not orthogonal to any edge of P.) The resulting oriented graph G(P,w) is useful in investigations of the numbers of faces of P.

Arne Brøndsted
An Introduction to Convex Polytopes
Springer-Verlag
New York 1983

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" 3-540-90722-X —" — Berlin

A COMPARISON OF LATTICE THEORETIC APPROACHES TO FUZZY TOPOLOGY (abstract)

Patrik Eklund

A *fuzzy lattice* L is a complete, completely distributive lattice with order reversing involution. For a set X an $(L-)$*fuzzy set* is a mapping $\mu : X \to L$. A *Hutton topology* on a fuzzy lattice L is a subcollection τ of L containing 0, 1 and being closed under finite infima, arbitrary suprema. The Hutton topologies (L,τ) form a category H where morphisms $\Phi : (L,\tau) \to (M,\sigma)$ are maps, denoted Φ^{-1}, from M to L preserving complement, arbitrary infima and suprema, and satisfying $\beta \in \sigma \Rightarrow \Phi^{-1}(\beta) \in \tau$. *Rodabaugh* has defined a category FUZZ of fuzzy topological spaces (X,L,τ), where X is a set, L is a fuzzy lattice, and τ is such that (L^X,τ) is a Hutton topology. Morphisms in FUZZ are given by $(f,\Phi) : (X,L,\tau) \to (Y,M,\sigma)$, where $f : X \to Y$, and Φ^{-1} is a map as above which satisfies $\nu \in \sigma \Rightarrow \Phi^{-1} \circ \nu \circ f \in \tau$. Category theoretic properties of H and FUZZ can be found in [Ek1].

The interaction between subcategories of H and FUZZ has, as far as known, not been investigated. As categorical relations the most important are isomorphism, equivalence and adjointness, the first being the strongest. Pretentious to constructing isomorphisms one must assume validity of the following

CONJECTURE. For each fuzzy lattice L there is a largest set X and a fuzzy lattice K such that L and K^X are fuzzy lattice isomorphic.

Let H_0 be the subcategory of H where objects remain the same but morphisms are enforced to preserve constants with respect to the Cartesian product given in the Conjecture. This subcategory is a natural restriction when involving the *Lowen* approach to fuzzy topology where constants are always open.

Besides H_0 consider the subcategory FUZZ_0 of FUZZ which contains all morphisms of FUZZ but where objects (X,L,τ) are such that L cannot be expressed as a Cartesian product of type K^X, K being a fuzzy lattice.

Using the conjecture one can establish an isomorphism between these subcategories. There is then a need to go through categorical properties of these subcategories. The properties, valid or not, will in many ways illustrate the advantages and disadvantages of H and FUZZ.

PROBLEM. Prove the Conjecture.

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Entropies of some random graphs

Ove Frank

Four kinds of occupancy models are considered for random distribution of equal or unequal objects into equal or unequal cells. The models are represented by randomly colored and labelled graphs. The entropies of these random graphs are evaluated and compared. Some possible applications are indicated.

"On the covering radius of cyclic linear codes and arithmetic codes"

by

Tor Helleseth
Forsvarets overkommando, Norge

The covering radius of a code is the smallest integer ρ such that the spheres of radius ρ around the codewords cover the whole space.

The problem of finding the covering radius for irreducible cyclic codes is shown to be related to Waring's problem in a finite field and to the theory of cyclotomic numbers.

Previously it has been known that the covering radius of a t-error-correcting BCH code equals $\rho=2t-1$ for $t=1,2$, and 3. For $t>3$ almost nothing has been known. The methods applied here lead to the bounds $2t-1 \leq \rho \leq 2t+1$ whenever $m \gg t > 3$. Recently Tietaväinen has improved the result further to $\rho \leq 2t$ for $m \gg t$. Whether $m \gg t$ implies $\rho=2t-1$ is still an open problem.

We have also applied similar methods to arithmetic codes. In particular new results have been obtained for the minimum distance and the covering radius of arithmetic codes generated by prime numbers.

Graph decomposition

Roland Häggkvist

The following theorem will be proved:

Theorem: Any tree T on m edges packs any $2m$-regular graph G of girth at most the diameter of T.

GENERALIZATIONS OF THE KORZHIK BOUND

Torleiv Kløve

Assume that symbols from the field $GF(q)$ are transmitted over a symmetric channel; the probability that a sent symbol is received correctly is $1-\varepsilon$ and the probability that it is transformed into a particular one of the other $q-1$ symbols is $\varepsilon/(q-1)$.

Let C be an (n,k,d) code, a linear code over $GF(q)$ of length n, dimension k, and minimum distance <u>at least</u> d. Let $P(C,t;\varepsilon)$ denote the probability that, after decoding, there remains an error that will not be detected, when the code is used to correct all error patterns with t or less errors, where $0 \le t \le d/2$. In particular $P(C,0;\varepsilon)$ is the probability of having an undetectable error when C is used purely as an error-detecting code.

Let $C(t) = \{ x \in GF(q)^n \mid d(x,c) \le t \text{ for some } c \in C \}$ where $d(.,.)$ denotes the Hamming distance, and let

$$A_{t,0},\ A_{t,1},\ \ldots\ ,A_{t,n}$$

denote the Hamming weight distribution of $C(t)$. Further let

$$F_t(z) = \sum_{j=d-t}^{n} A_{t,j}\, z^j.$$

Then

$$P(C,t;\varepsilon) = (1-\varepsilon)^n\, F_t\!\left(\frac{\varepsilon}{(1-\varepsilon)(q-1)}\right).$$

Define $P[q,n,k,t,\varepsilon]$ to be the minimum of $P(C,t;\varepsilon)$ over all $(n,k,2t+1)$ codes C over $GF(q)$ if such codes exist, and define $P[q,n,k,t,\varepsilon]=\infty$ otherwise.

Korzhik showed that for all n, k, and ε,

$$P[2,n,k,0,\varepsilon] \le 2^{k-n}(1-(1-\varepsilon)^k).$$

This result has been strengthened in various ways by Levenshtein, Kasami et al., and Wolf et.al. We have studied the underlying idea and obtained the following general bounds on $P[q,n,k,t,\varepsilon]$.

<u>Theorem</u> Let X be any non-empty set of $(n,k,2t+1)$-codes over $GF(q)$. Let

$$\alpha_j = \frac{1}{\#X} \sum_{\substack{x \in GF(q)^n \\ w(x)=j}} \#\{ C \in X \mid x \in C(t) \}.$$

Then

$$P[q,n,k,t,\varepsilon] \le \sum_{j=t+1}^{n} \alpha_j \left(\frac{\varepsilon}{q-1}\right)^j (1-\varepsilon)^{n-j}.$$

<u>Theorem</u> Let K be an $(n,\kappa,2t+1)$ code over $GF(q)$. Then

$$P[q,n,k,t,\varepsilon] \le \frac{q^k-1}{q^\kappa-1} P(K,t;\varepsilon) \quad \text{for all } k \le \kappa.$$

On harmonic conjugates in algebraic matroids

Bernt Lindström

By an analogy in classical projective geometry I will define harmonic points and harmonic conjugates of a point a with respect to two other points, x and y, in an algebraic matroid of rank 2. Three examples of harmonic conjugates are $\{x+y,\ x-y\}$, $\{xy,\ x/y\}$ and $\left\{\frac{x+y}{1+xy},\ \frac{x-y}{1-xy}\right\}$. It seems likely that further non-trivial examples can be obtained with the aid of trigonometric and elliptic functions. I do not know whether every point on an "algebraic line" has a harmonic conjugate, but I guess that this is not the case.

I can prove a sufficient condition, which depends on a lemma by Ingleton and Main (1975) used by them to prove that Vamos' matroid is non-algebraic. Finally I will mention a simple example of a non-algebraic matroid of rank 3.

Determination of Transfer Functions for a Class of Electrical Networks by a Graph Theoretic Algorithm.

Frank Nielsen.

Abstract. One of the first contributions to Graph Theory is Kirchhoff's paper from 1847. In this paper it is described how the currents in a resistive network driven by a voltage generator can be determined by a graph theoretic method. In the lecture a generalization (mainly due to Davies) of Kirchhoff's method to a more general class of networks is described. The networks considered contain resistors, capacitors, inductors and operational amplifiers. Mathematical models of these elements will be described, and no prior knowledge of network theory is assumed.

Öystein J. Rödseth: On Sums of Sets of Residue Classes.

Let $a_1,\ldots,a_k$ be distinct residues modulo a prime p, and let s denote the number of distinct residues of the form $a_i + a_j$, $i \neq j$. Is $s \geq \min(p, 2k-3)$? This is an old problem, apparently due to Erdös and Heilbronn ([4]?, cf. [3, p.95], [6, p.73]), and as far as I know, it is still open.

Burde [2] uses linear algebra to give a (relatively) simple proof of the fact that for a fixed k, we have $s \geq 2k-3$ for all but finitely many primes p. But a more precise result is known (cf. [1]): If $p > 6\cdot 4^{k-4}$, then $s \geq 2k-3$. It is, however, an easy consequence of a (deep) result of Freiman [5, p.93] that there exists an absolute constant c, such that $s \geq 2k-3$ if $p > ck$.

By taking into consideration the number of representations of a residue x as $x = a_i + a_j$, a result of Pollard [7] can be used to prove that we always have $s \geq \min(p, 2k-(4k+1)^{1/2})$. This result also holds when p is replaced by an arbitrary positive integer n, if $\gcd(a_i - a_j, n) = 1$ for $i \neq j$ (cf. [8]).

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Foredrag ved Nordiskt Symposium II i kombinatorik 2-4 maj 1984

Carsten Thomassen

Titel: Directed graphs of large girth.

Abstract: We give a precise description of the directed graphs of order n and girth at least $\frac{2}{3}n$. This can be used, among other things, to calculate, in polynomial time, the cycle length distribution of a directed graph of large girth.

CHARACTER SUMS AND BLOCK CODES

A. Tietäväinen

Let q be a prime power, F the field of q elements, d (≥ 2) a factor of $q-1$, and χ a multiplicative character of F of order d. Define

$$S(d,r,q) = \max \Big| \sum_{\alpha \in F} \chi(f(\alpha)) \Big|$$

where the maximum is taken over all elements f of $F[x]$ which are not d^{th} powers and have exactly r distinct zeros. For $r < q^{\frac{1}{2}}+1$ we have Weil's upper bound (see [1, pp. 43 and 46])

$$S(d,r,q) \leqq (r-1)q^{\frac{1}{2}} \tag{1}$$

and for $r \geqq q^{\frac{1}{2}}+1$ the trivial upper bound

$$S(d,r,q) \leqq q. \tag{2}$$

It is natural to ask how good lower bounds there may be. In this talk I describe a block code method presented in the papers [2], [3] and [4] and show that

$$S(d,2s,q) = q \text{ if } s \geqq q/\log_d q + 2$$

and

$$S(d,r,q) \gtrsim c(r)q^{\frac{1}{2}} \text{ when } q \to \infty \text{ where } c(r) \sim \sqrt{\frac{r}{2}} \text{ when } r \to \infty.$$

Thus the trivial upper bound (2) is best possible when r is large and we also see that in Weil's inequality (1) the exponent $\frac{1}{2}$ cannot be improved and the coefficient $r-1$ cannot be replaced by a number essentially smaller than $\sqrt{\frac{r}{2}}$.

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Helge Tverberg: On Schmerl's effective version of Brooks' theorem.

Abstract.

Let v_1, v_2, ..., be the vertices of a countably infinite graph G. G is said to be effectively k-colourable if there is a k-colouring for which an algorithm exists to compute the colour of v_n. Schmerl (Can. J. Math. 34 (1982), 1036-1046) proved that if G satisfies Brooks' condition for k-colourability, and if there is an algorithm to compute the neighbours of v_n, then G is effectively k-colourable. In this talk it will be explained how this result can be obtained from a recent proof of Brooks' theorem (Tverberg, Math. Scand. 52 (1983), 37-40).

<u>Cycles in graphs</u>

Preben Dahl Vestergaard

<u>Abstract</u>:

The problem of covering the edges of a graph with a collection of distinct circuits such that each edge of the graph is contained in exactly k circuits is considered. For k=1 we are considering Eulerian graphs, for k=2 we are considering P.D. Seymour's double cover conjecture. For k=2 the problem is reduced to 3-regular graphs.

TOLERANCE RELATIONS IMPOSED ON SET-ALGEBRAS

R. Wasén Alphyddan, Gärdala
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Z. Pawlak in Warsaw has developed a notion of approximation of information in attribute-based information systems. Mathematically, in a set U with an equivalence relation $\sim$ on U he developed a theory to approximate relations of power-sets $\mathcal{P}(U)$ modulo the equivalence relation $\sim$.

Tolerance relations are generalizations of equivalence relations and are in a sense closer to the notion of an approximation. We have therefore generalized Pawlak's theory to include the case of tolerance relations. Combinatorially things are considerably more complicated, but may also be more interesting. Concerning theorems about with certain extremal properties as what concerns approximations this holds true.

Pawlaks concepts have been efficiently implemented in a computer environment and applied in different contexts. Efficient implementations of our generalization will depend on the mathematical skill in designing the algorithms involved, on the care that is taken to the specific computer environment in question and possible restrictions on the tolerance relations.

Sände Prof. som [illegible].

Sända särtryck.

Sändt 16.5.84.
Särtryck/kopia